



Faculty of Engineering and Technology
Department of Electrical and Computer Engineering

ENCS4310
DIGITAL SIGNAL PROCESSING (DSP)
Assignment No (1)

Prepared by:

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Section: 3

Supervised by:

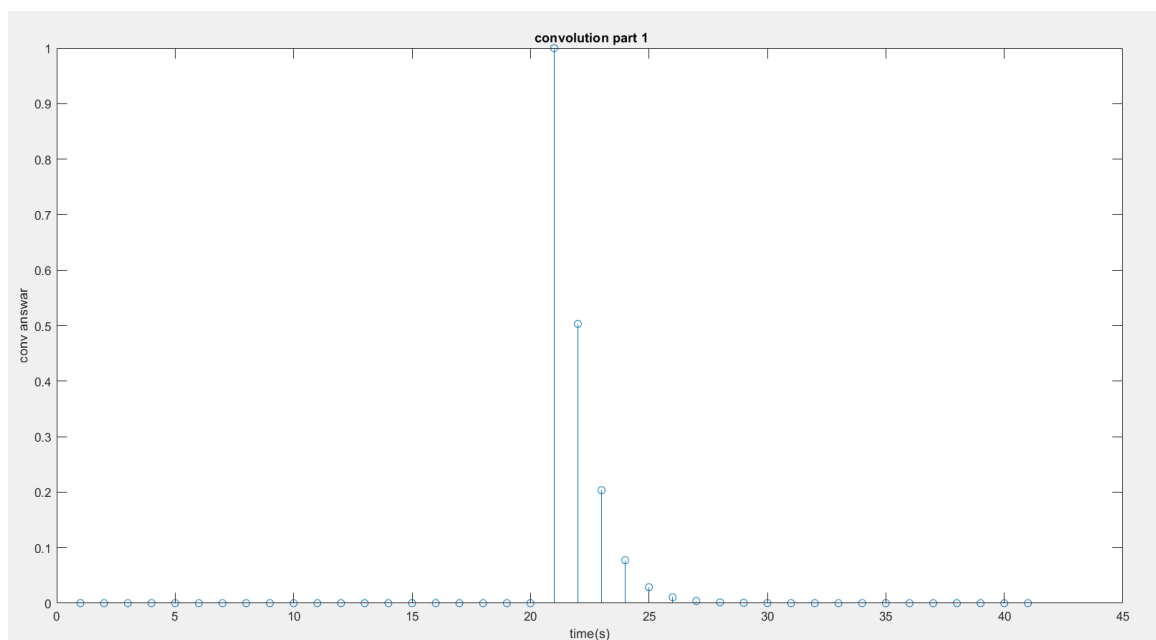
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Date: 20/11/2020

Question 1: convolution

$$(a) \quad x(n) = \exp(-n)u(n), \quad h(n) = \exp(-2n)u(n)$$

```
nour.m x +
1 - n = -10:1:10;
2 - x=exp(-1.*n).*heaviside(n) ;
3 - h= exp(-2.*n).*heaviside(n);
4 - y = conv(x,h);
5 - figure(1)
6 - stem(y);
7 - title(' convolution part 1');
8 - xlabel('time(s)');
9 - ylabel('conv answar');
```



The solution:

$$\textcircled{a} \quad x[n] = e^{-n} u[n]$$

$$h[n] = e^{-2n} u[n]$$

$$\hookrightarrow x(e^{j\omega}) = \sum_{n=0}^{\infty} x[n] e^{j\omega n}$$

$$= \sum_{n=0}^{\infty} (e^{-1} e^{-j\omega})^n = \frac{1}{1 - e^{-1} e^{-j\omega}} \quad ; \quad z = e^{j\omega}$$

$$\Rightarrow a^n u[n] \rightarrow \frac{1}{1 - a z^{-1}}$$

$$\hookrightarrow X(z) = \frac{1}{1 - e^{-1} z^{-1}}$$

$$\hookrightarrow H(z) = \frac{1}{1 - e^{-2} z^{-1}}$$

$$Y[n] = x[n] * h[n]$$

$$Y(z) = X(z) \cdot H(z)$$

$$Y(z) = \frac{1}{1 - e^{-1} z^{-1}} \cdot \frac{1}{1 - e^{-2} z^{-1}}$$

$$\Rightarrow \frac{1}{(1 - e^{-1} z^{-1})(1 - e^{-2} z^{-1})} = \frac{A}{1 - e^{-1} z^{-1}} + \frac{B}{1 - e^{-2} z^{-1}}$$

$$A(1 - e^{-2} z^{-1}) + B(1 - e^{-1} z^{-1}) = 1$$

$$\text{when } \boxed{z^{-1} = e^2} \Rightarrow B(1 - e) = 1 \Rightarrow B = \frac{1}{1 - e}$$

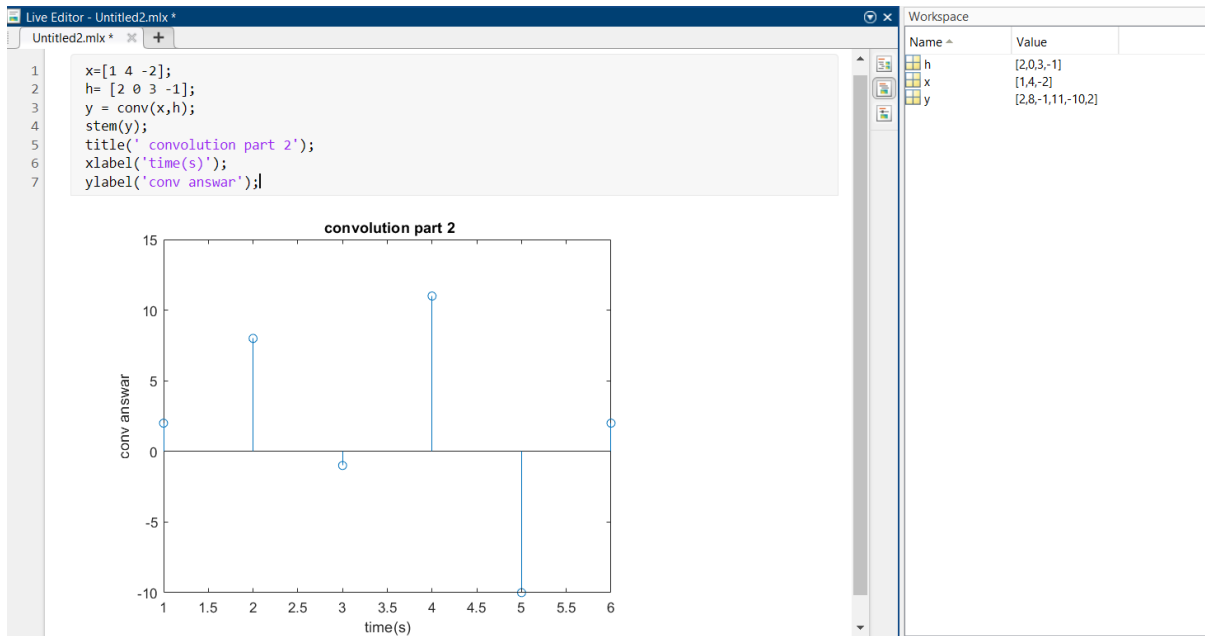
$$\text{when } \boxed{z^{-1} = e} \Rightarrow A(1 - e^{-1}) = 1 \Rightarrow A = \frac{1}{1 - e^{-1}}$$

$$\Rightarrow Y(z) = \frac{1}{1 - e^{-1}} \cdot \frac{1}{1 - e^{-2} z^{-1}} + \frac{1}{1 - e} \cdot \frac{1}{1 - e^{-1} z^{-1}}$$

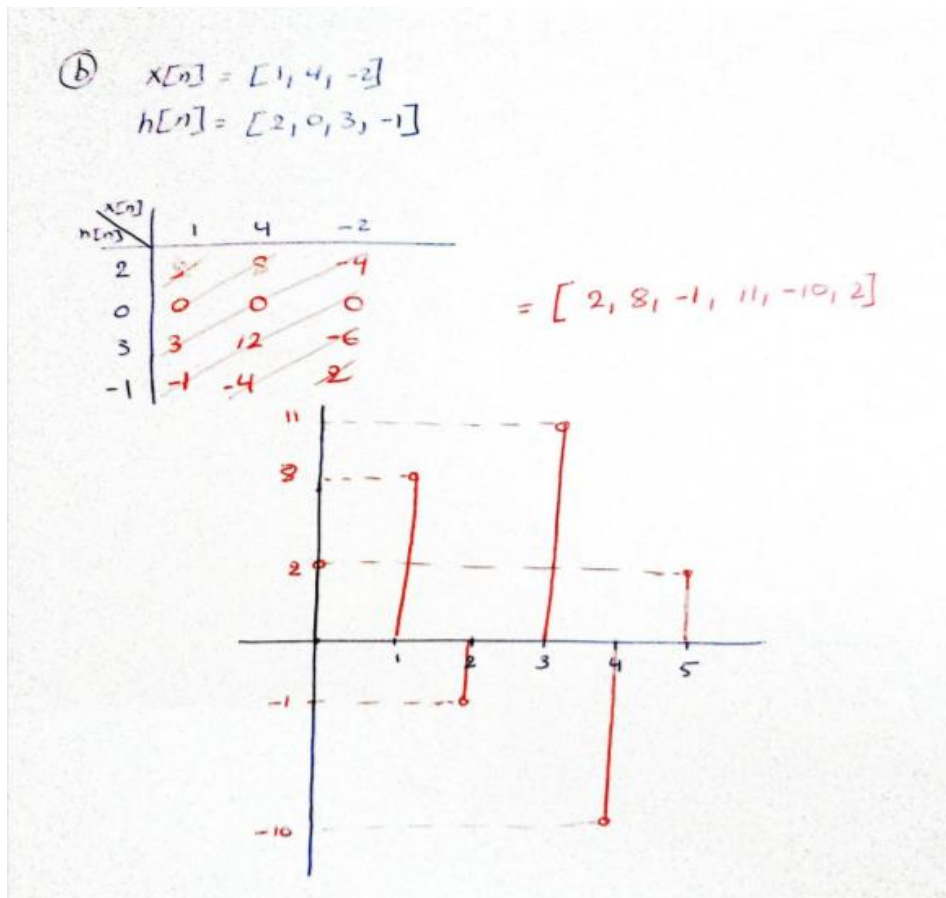
$$\Rightarrow Y[n] = \frac{1}{1 - e^{-1}} \cdot e^{-n} u[n] + \frac{1}{1 - e} e^{-2n} u[n]$$

$$Y[n] = (1.58197) e^{-n} u[n] - (0.58197) e^{-2n} u[n]$$

(b) $x(n) = [1, 4, -2], h(n) = [2, 0, 3, -1]$



The solution:



Question 2: Delay and Echo

The following example showed how to load a wav file and play it with Matlab.

Consider a music piece is being played in a round theater where the orchestra is in the middle of two concentric circles and the walls on one half side are at a radial distances of 17.15 meters (corresponding to the inner circle) and 34.3 meters (corresponding to the outer circle) on the other side from the orchestra. The speed of sound is 343 meters/sec.

The recorder is at the same location as the orchestra. The recorded signal is the original plus the echoes from the two walls, as

$$y(n) = x(n) + 0.8x(n - N_1) + 0.6x(n - N_2) \quad (1)$$

- What is the impulse response of the above system?
- If the sampling rate is 44.1 kbps, Find the values of N_1 and N_2 .
- Implement the system by using convolution. Play the sound and compare it to the original piece. Try the system on both female voice.wav and music.wav.
- Adjust the amplitude of the two reflected signals and see how that affect the sound.

Q2 := 1)

a) $y[n] = x[n] + 0.8x[n - N_1] + 0.6x[n - N_2]$

$$Y(z) = X(z) + 0.8X(z)z^{-N_1} + 0.6X(z)z^{-N_2}$$

$$Y(z) = X(z)[1 + 0.8z^{-N_1} + 0.6z^{-N_2}]$$

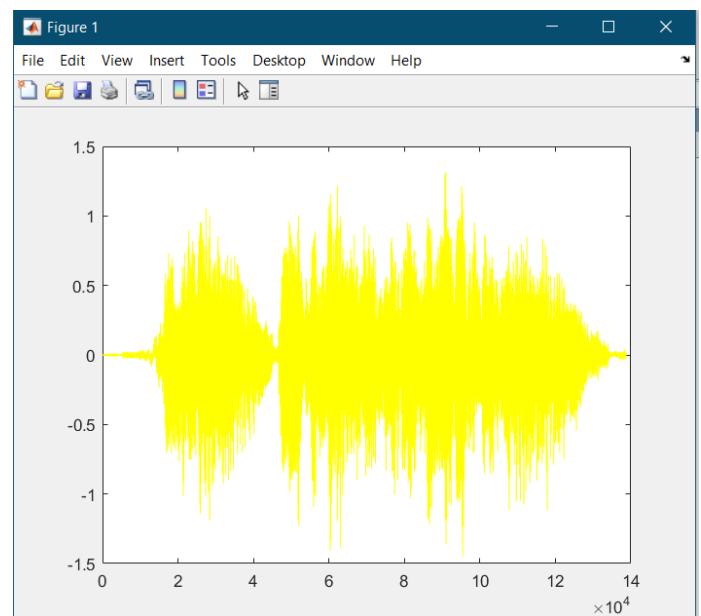
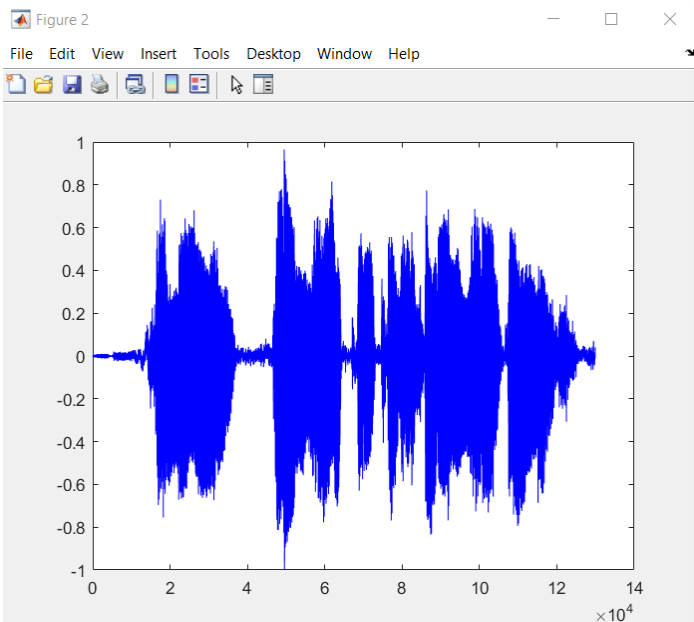
$$H(z) = \frac{Y(z)}{X(z)} = 1 + 0.8z^{-N_1} + 0.6z^{-N_2}$$

$$h[n] = \delta[n] + 0.8\delta[n - N_1] + 0.6\delta[n - N_2]$$

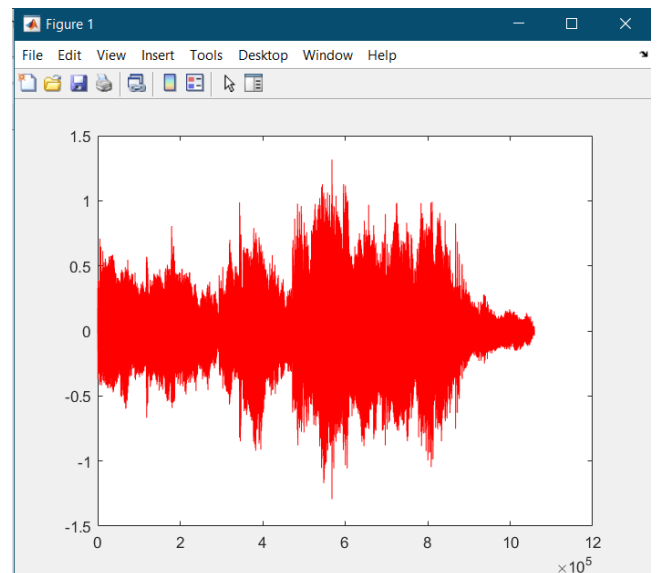
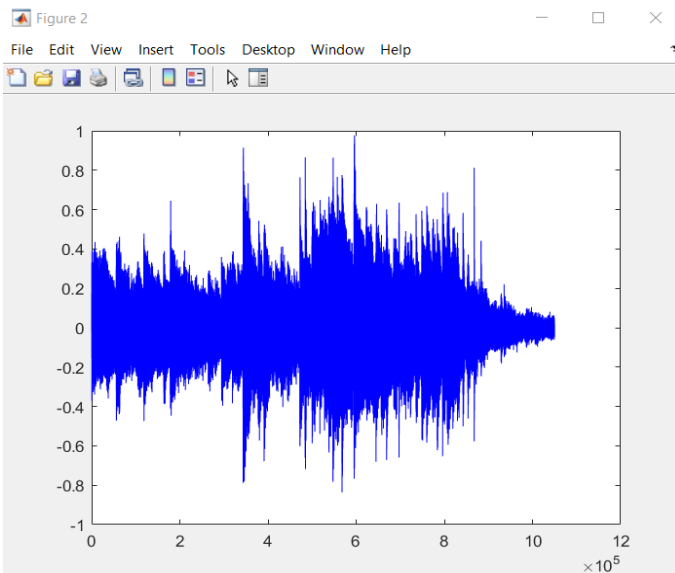
b) $N_1 = T_1 = \frac{(17.15)(2)}{343} \cdot T_s = 4410$

$$N_2 = T_2 = \frac{(34.3)(2)}{343} \cdot T_s = 8820$$

```
1
2 - [data,Fs] =audioread('female_voice.wav');
3 - x_left = data(:,1).';
4 - x_right= data(:,2).';
5 - Ts=1/Fs;
6 - x_left =x_left/ max([ abs(x_left), abs(x_right)]);
7 - x_right =x_right/ max([ abs(x_left), abs(x_right)]);
8 - y=[x_left.', x_right.'];
9 - sound(y,Fs);
:0 - Fs=44100;
:1 - num=[1,zeros(1,4410),0.8,zeros(1,8820-4410),0.6];
:2 - den=[1];
:3 - y_left =conv(num, x_left);
:4 - y_right =conv(num,x_right);
:5 - pause(0);
:6 - z=[y_left.', y_right.'];
:7 - sound(z,Fs);
:8 - figure(1);
:9 - plot(z,'y')
:0 - figure(2);
:1 - plot(y,'b')
```



```
1
2 - [data,Fs] =audioread('music.wav');
3 - x_left = data(:,1).';
4 - x_right= data(:,2).';
5 - Ts=1/Fs;
6 - x_left =x_left/ max([ abs(x_left), abs(x_right)]);
7 - x_right =x_right/ max([ abs(x_left), abs(x_right)]);
8 - y=[x_left.', x_right.'];
9 - sound(y,Fs);
10 - Fs=44100;
11 - num=[1,zeros(1,4410),0.8,zeros(1,8820-4410),0.6];
12 - den=[1];
13 - y_left =conv(num, x_left);
14 - y_right =conv(num,x_right);
15 - pause(0);
16 - z=[y_left.', y_right.'];
17 - sound(z,Fs);
18 - figure(1);|
19 - plot(z,'r')
20 - figure(2);
21 - plot(y,'b')
22
```



- The sound becomes more realistic as the amplitude of the two reflected signals is adjusted.

Question 3: Impulse response

A discrete-time LTI system is represented by the difference equation

$$y(n) = 0.15y(n-2) + x(n) \quad (2)$$

(a) Write a Matlab code to find the impulse response of the system by setting $x(n) = \delta(n)$, and then recursively calculate the value of $y(n)$ by following the difference equation (Hint: The impulse response has infinite length. You only need to find the first 15 values of $h(n)$).

(b) Use the Matlab function `filter()` to get the impulse response. Compare the result with part (a). (Hint: use help `filter` to learn about the function `filter`. You can set the input vector as $x(n) = [1, \text{zeros}(1, 14)]$)

```

1 - n = 0:14;
2 - xn = (n==0); %Impulse Function.
3 - N = length(n);
4 - % (a) :
5 - y_1 = 0; % initial condition = 0
6 - y1 = 0; % initial condition = 0
7 - yn(2) = 0.15*y1+xn(2);
8 - yn(1) = 0.15*y_1+xn(1); |
9 - for k=3:N
10 - yn(k) = 0.15*yn(k-2)+xn(k)
11 - end
12 - subplot(211)
13 - stem(n,yn)
14 - title(' Matlab code to find the impulse response of the system by setting x(n) = delta(n)')
15 -
16 - % (b) Using filter() Command:
17 - a = [1 0 0 0];
18 - b = [1 0 -0.15];
19 - w=(n==0);
20 - yn = filter(a,b,w);
21 - subplot(212)
22 - stem(n,yn)
23 - title('Matlab function filter() to get the impulse response')
24 -

```



Question 4: Z-transform

1. (a) Read the help file of the Matlab function **residuez()** and **filter()**.

(b) Read, repeat, and understand the following example, which performs inverse Z-transform of the following signal with partial fraction expansion.

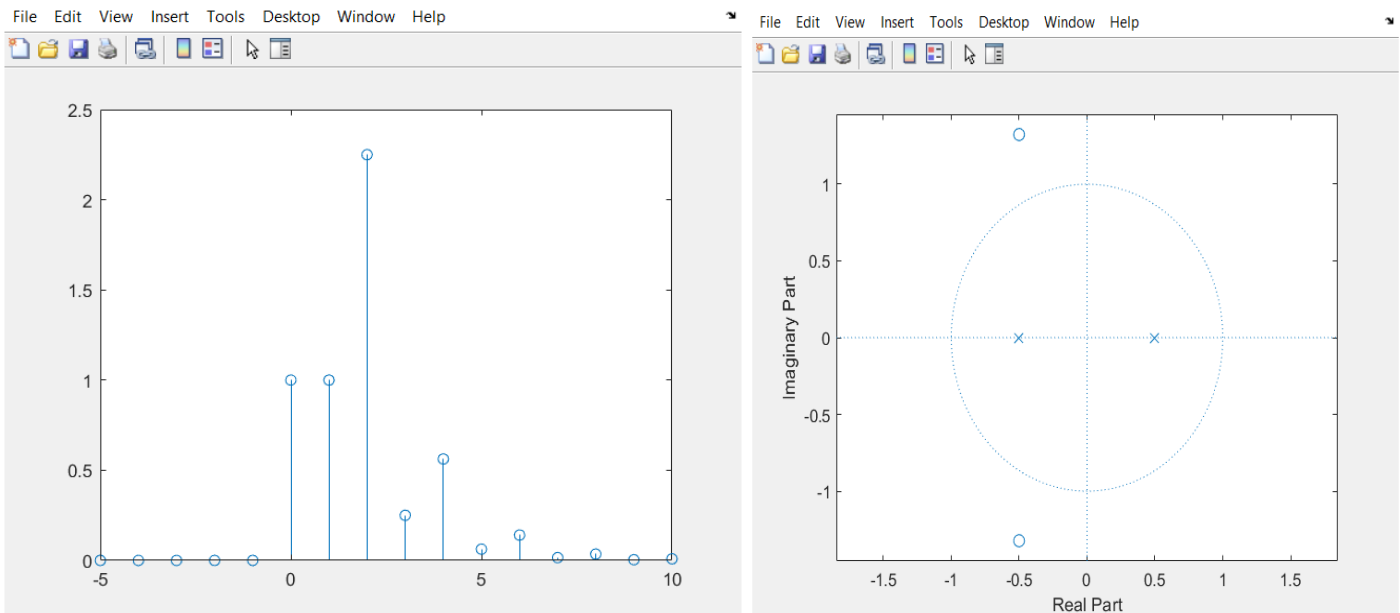
$$X(z) = \frac{1 + z^{-1} + 2z^{-2}}{1 - 0.25z^{-2}}$$

```

1 - uimpulse = inline('double (n==0)');
2 - ustep = inline('double (n>=0)');
3 - b= [1, 1, 2];
4 - a = [1 , 0 , -0.25];
5 - [r, p, k] = residuez(b, a)
6 - z = roots(b);
7 - figure(1);
8 - zplane (b, a);
9 - n = [-5:10];
10 - xn = 5.5*(0.5).^n.*ustep(n)+3.5*(-0.5).^n.*ustep(n)-8*uimpulse(n);
11 - figure(2);
12 - stem (n, xn);
13 - hold on ;
14 - xn_2 = filter(b, a, uimpulse(n) );
15 - stem(n, xn_2, 'r+:-');

```

graph:



The solution:

$$X(z) = \frac{1 + z^{-1} + 2z^{-2}}{1 - 0.25z^{-2}}$$

$$\frac{1 - 0.25z^{-2} \left[\begin{array}{l} -8 \\ 1 + z^{-1} + 2z^{-2} \\ + +8 + 2z^{-1} \end{array} \right]}{9 + z^{-1}}$$

$$-8 + \frac{9z^{-1}}{1 - 0.25z^{-2}}$$

$$-8 + \frac{A}{1 - \frac{1}{2}z^{-1}} + \frac{B}{1 + \frac{1}{2}z^{-1}} = \frac{1 + z^{-1} + 2z^{-2}}{1 - 0.25z^{-2}}$$

$$-8(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{2}z^{-1}) + A(1 + \frac{1}{2}z^{-1}) + B(1 - \frac{1}{2}z^{-1}) = 1 + z^{-1} + 2z^{-2}$$

$$\text{When } z^{-1} = 2 \Rightarrow A(2) = 1 + 2 + 8 \Rightarrow \boxed{A = \frac{11}{2}}$$

$$\text{When } z^{-1} = -2 \Rightarrow B(2) = 1 - 2 + 8 \Rightarrow \boxed{B = \frac{7}{2}}$$

$$\Rightarrow -8 + \frac{11/2}{1 - \frac{1}{2}z^{-1}} + \frac{7/2}{1 + \frac{1}{2}z^{-1}} = X(z)$$

$$X[n] = -8\delta[n] + \frac{11}{2} \left(\frac{1}{2}\right)^n u[n] + \frac{7}{2} \left(-\frac{1}{2}\right)^n u[n]$$

2. (a) Find the inverse Z-transform of the following signals by using partial fraction expansion.
 (b) Plot the poles and zeros on the z-plane.
 (c) Plot the time domain signal $x(n)$ with both of the two methods in the example

$$X(z) = \frac{8 - 4z^{-1}}{z^{-2} + 6z^{-1} + 8} \quad (2)$$

$$X(z) = \frac{z^3 + 1}{z^3 - z^2 - z - 2} \quad (3)$$

$$X(z) = \frac{0.6z^2}{z^3 - 0.5z^2 - 0.25z + 0.125} \quad (4)$$

Q4/2-A

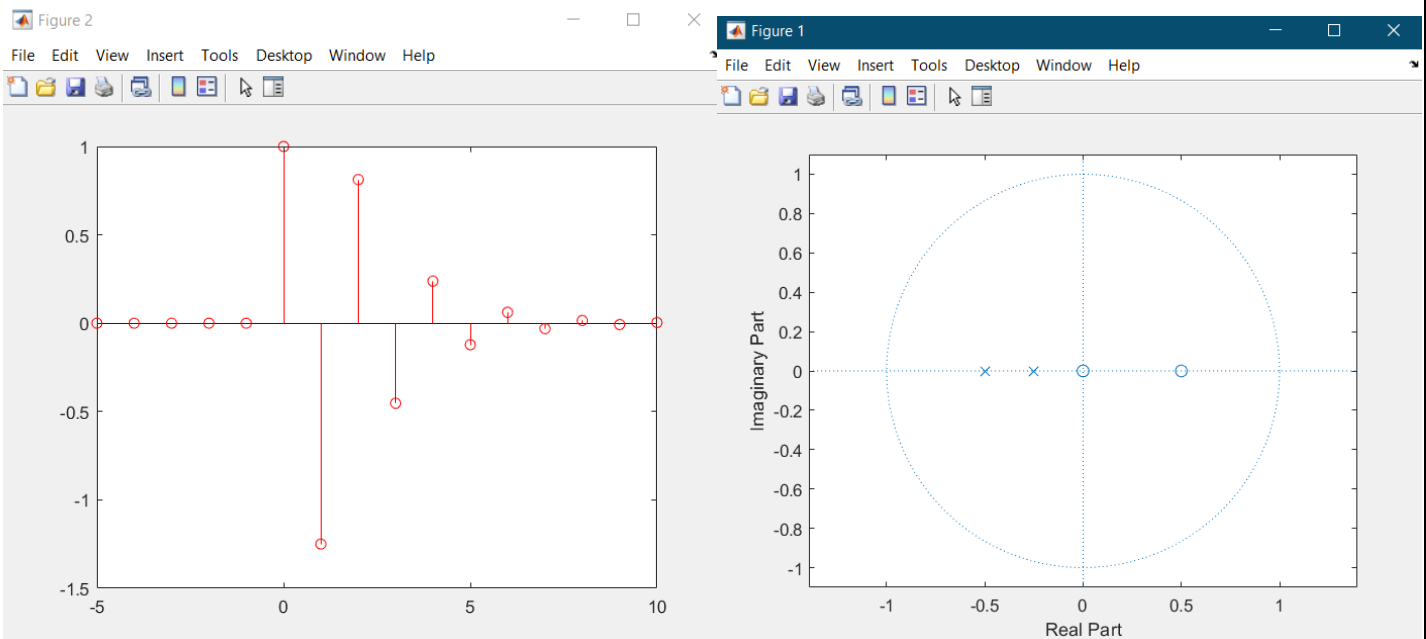
The solution:

1.

```

1 - uimpulse = inline('double (n==0)');
2 - ustep = inline('double (n>=0)');
3 - b= [8, -4, 0];
4 - a = [8 , 6 , 1];
5 - [r, p,k] = residuez(b, a)
6 - z = roots(b);
7 - figure(1);
8 - zplane (b, a);
9 - n = [-5:10];
10 - xn = r(1)*(p(1)).^n.*ustep(n)+r(2)*(p(2)).^n.*ustep(n)+k*uimpulse(n);
11 - figure(2);
12 - stem (n, xn);
13 - hold on ;
14 - xn_2 = filter(b, a, uimpulse(n));
15 - stem(n, xn_2, 'r');

```

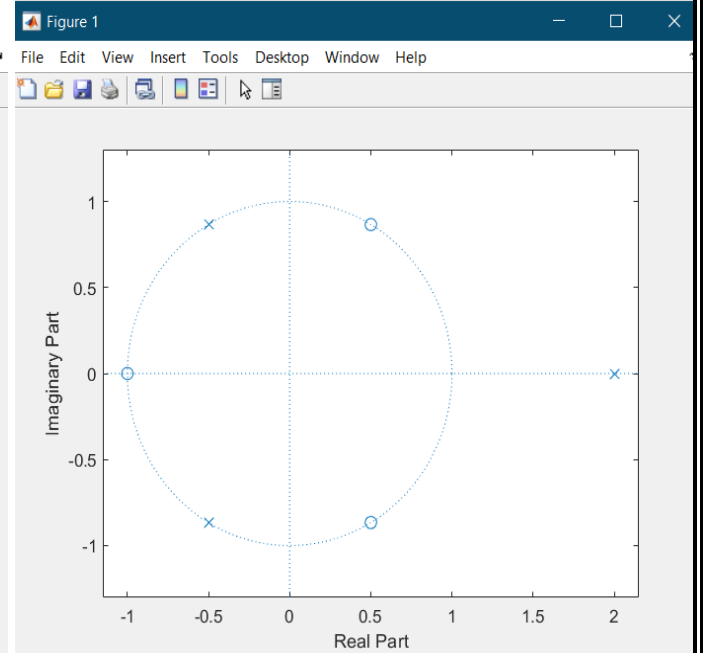
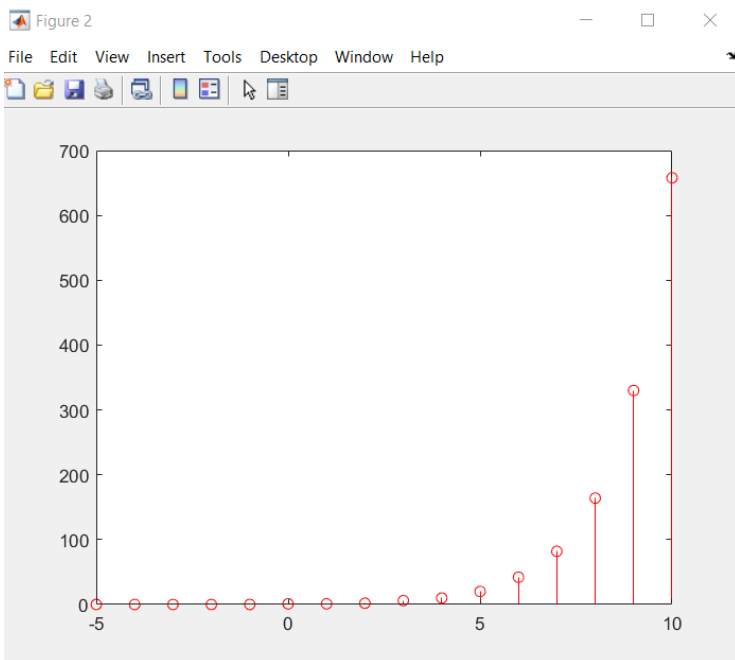


2.

```

1 - uimpulse = inline('double (n==0)');
2 - ustep = inline('double (n>=0)');
3 - b= [1,0,0,1];
4 - a = [1,-1,-1,-2];
5 - [r, p, k] = residuez(b, a)
6 - z = roots(b);
7 - figure(1);
8 - zplane (b, a);
9 - n = [-5:10];
10 - xn = r(1)*(p(1)).^n.*ustep(n)+r(2)*(p(2)).^n.*ustep(n)+r(3)*(p(3)).^n.*ustep(n)+k*uimpulse(n);
11 - figure(2);
12 - stem (n, xn);
13 - hold on ;
14 - xn_2 = filter(b, a, uimpulse(n));
15 - stem(n, xn_2, 'r');|

```

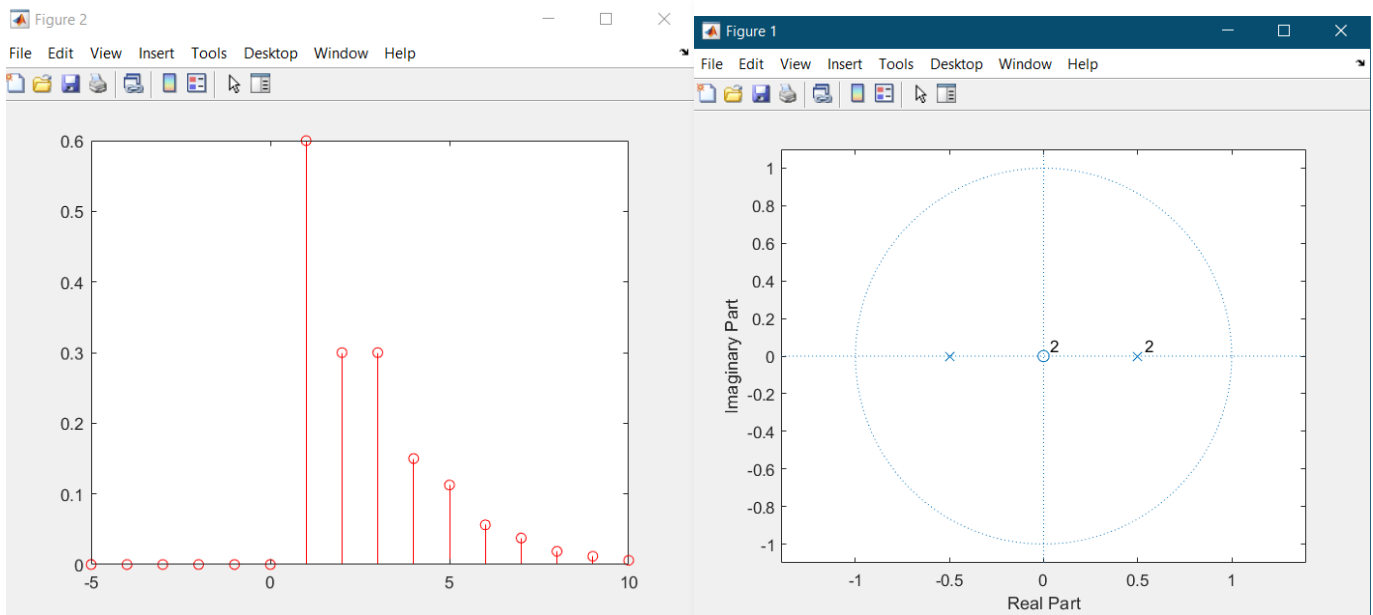


3.

```

1 - uimpulse = inline('double (n==0)');
2 - ustep = inline('double (n>=0)');
3 - b= [0,0.6,0,0];
4 - a = [1,-0.5,-0.25,0.125];
5 - [r, p, k] = residuez(b, a)
6 - z = roots(b);
7 - figure(1);
8 - zplane (b, a);
9 - n = [-5:10];
10 - xn = r(1)*(p(1)).^(n).*ustep(n)+r(2)*(p(2)).^(n).*ustep(n+1)+r(3)*(p(3)).^n.*ustep(n)+k*uimpulse(n);
11 - figure(2);
12 - stem (n, xn);
13 - hold on ;
14 - xn_2 = filter(b, a, uimpulse(n) );
15 - stem(n, xn_2, 'r');
16 |

```



The solution :

$$X(z) = \frac{8 - 4z^{-1}}{z^{-2} + 6z^{-1} + 8} = \frac{8 - 4z^{-1}}{(z^{-1} + 2)(z^{-1} + 4)}$$

$$\frac{A}{z^{-1} + 2} + \frac{B}{z^{-1} + 4} = \frac{8 - 4z^{-1}}{(z^{-1} + 2)(z^{-1} + 4)}$$

$$A(z^{-1} + 4) + B(z^{-1} + 2) = 8 - 4z^{-1}$$

$$\text{when } \boxed{z^{-1} = -4} \Rightarrow B(-2) = 8 - 4(-4)$$

$$B(-2) = 24 \quad \boxed{B = -12}$$

$$\text{when } \boxed{z^{-1} = -2} \Rightarrow A(2) = 8 - 4(-2)$$

$$A(2) = 16 \Rightarrow \boxed{A = 8}$$

$$\Rightarrow \frac{8}{z^{-1} + 2} + \frac{-12}{z^{-1} + 4} = X(z)$$

$$\frac{8}{2(1 + \frac{1}{2}z^{-1})} + \frac{-12}{4(1 + \frac{1}{4}z^{-1})} = X(z)$$

$$X(z) = \frac{4}{1 + \frac{1}{2}z^{-1}} - \frac{3}{1 + \frac{1}{4}z^{-1}}$$

$$X[n] = 4\left(\frac{-1}{2}\right)^n u[n] - 3\left(\frac{-1}{4}\right)^n u[n]$$

3.

$$X(z) = \frac{0.6 z^{-2}}{z^3 - 0.5 z^2 - 0.25 z + 0.125}$$

$$= \frac{0.6 z^{-2}}{z^2(z-0.5) - 0.25(z-0.5)}$$

$$= \frac{0.6 z^{-2}}{(z-0.5)(z-0.5)(z+0.5)}$$

$$= \frac{A}{z+0.5} + \frac{B}{z-0.5} + \frac{C}{(z-0.5)^2}$$

$$= A(z-0.5)^2 + B(z-0.5)(z+0.5) + C(z+0.5) = 0.6z^2$$

$$\text{when } z = 0.5 \Rightarrow C(1) = (0.6)(0.5)^2 \Rightarrow C = 0.15$$

$$\text{when } z = -0.5 \Rightarrow A(-1)^2 = 0.6(-0.5)^2 \Rightarrow A = 0.15$$

$$\text{when } z = 1 \Rightarrow A\left(\frac{1}{2}\right)^2 + B\left(\frac{1}{2}\right)\left(\frac{3}{2}\right) + C\left(\frac{3}{2}\right) = 0.6(1)$$

$$\therefore \boxed{B = 0.45}$$

$$\Rightarrow \frac{0.15}{z\left(1 + \frac{1}{2}z^{-1}\right)} + \frac{0.45}{z\left(1 - \frac{1}{2}z^{-1}\right)} + \frac{0.15}{z^2\left(1 - \frac{1}{2}z^{-1}\right)^2}$$

$$= 0.15 \left(\frac{1}{2}\right)^{n-1} u[n-1] + 0.45 \left(\frac{1}{2}\right)^{n-1} u[n-1] + 0.3 \left(\frac{1}{2}\right)^{n-1} (n-1) u[n-1]$$

$$\frac{0.15 z^{-2}}{z\left(1 - \frac{1}{2}z^{-1}\right)^2} = \frac{0.15 z^{-1}}{0.5} \left[\frac{\left(\frac{1}{2}\right)z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)^2} \right]$$

$$= 0.3 z^{-1} \left[\frac{0.5 z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)^2} \right] = 0.3 \left(\frac{1}{2}\right)^{n-1} (n-1) u[n-1]$$

Appendix:

Q1/A

```
n = -10:1:10;
x=exp(-1.*n).*heaviside(n) ;
h= exp(-2.*n).*heaviside(n);
y = conv(x,h);
figure(1)
stem(y);
title('convolution part 1');
xlabel('time(s)');
ylabel('conv answer');
```

Q1/B

```
x=[1 4 -2];
h= [2 0 3 -1];
y = conv(x,h);
stem(y);
title('convolution part 2');
xlabel('time(s)');
ylabel('conv answer');
```

Q2/

```
[data,Fs] =audioread('music.wav');
x_left = data(:,1).';
x_right= data(:,2).';
Ts=1/Fs;
x_left =x_left/ max([ abs(x_left), abs(x_right)]);
x_right =x_right/ max([ abs(x_left), abs(x_right)]);
y=[x_left.', x_right.'];
sound(y,Fs);
Fs=44100;
num=[1,zeros(1,4410),0.8,zeros(1,8820-4410),0.6];
den=[1];
y_left =conv(num, x_left);
y_right =conv(num,x_right);
pause(0);
z=[y_left.', y_right.'];
sound(z,Fs);
figure(1);
plot(z,'r')
figure(2);
plot(y,'b')
```



```
[data,Fs]=audioread('female_voice.wav');
x_left = data(:,1).';
x_right= data(:,2).';
Ts=1/Fs;
x_left =x_left/ max([ abs(x_left), abs(x_right)]);
x_right =x_right/ max([ abs(x_left), abs(x_right)]);
y=[x_left.', x_right.'];
sound(y,Fs);
Fs=44100;
num=[1,zeros(1,4410),0.8,zeros(1,8820-4410),0.6];
den=[1];
y_left =conv(num, x_left);
y_right =conv(num,x_right);
pause(0);
z=[y_left.', y_right.'];
sound(z,Fs);
figure(1);
plot(z,'y')
figure(2);
plot(y,'b')
```

Q3

```
n = 0:14;
xn =(n==0); %Impulse Function.
N = length(n);
% (a) :
y_1 = 0; % initial condition = 0
y1 = 0; % initial condition = 0
yn(2) = 0.15*y1+xn(2);
yn(1) = 0.15*y_1+xn(1);
for k=3:N
yn(k) = 0.15*yn(k-2)+xn(k)
end
subplot(211)
stem(n,yn)
title(' Matlab code to find the impulse response of the system by setting x(n) = delta(n)')
```

% (b) Using filter() Command:

```
a = [1 0 0 0];
b = [1 0 -0.15];
w=(n==0);
yn = filter(a,b,w);
subplot(212)
stem(n,yn)
title('Matlab function filter() to get the impulse response')
```

Q4/A

```
uimpulse = inline('double (n==0)');
ustep = inline('double (n>=0)');
b= [1, 1, 2];
a = [1 , 0 , -0.25];
[r, p, k] = residuez(b, a)
z = roots(b);
figure(1);
zplane (b, a);
n = [-5:10];
xn = 5.5*(0.5).^n.*ustep(n)+3.5*(-0.5).^n.*ustep(n)-8*uimpulse(n);
figure(2);
stem (n, xn);
hold on ;
xn_2 = filter(b, a, uimpulse(n) );
stem(n, xn_2, 'r+:-');
```

Q4/B

1.

```
uimpulse = inline('double (n==0)');
ustep = inline('double (n>=0)');
b= [8, -4, 0];
a = [8 , 6 , 1];
[r, p,k] = residuez(b, a)
z = roots(b);
figure(1);
zplane (b, a);
n = [-5:10];
xn = r(1)*(p(1)).^n.*ustep(n)+r(2)*(p(2)).^n.*ustep(n)+k*uimpulse(n);
figure(2);
stem (n, xn);
hold on ;
xn_2 = filter(b, a, uimpulse(n));
stem(n, xn_2, 'r');
```

2.

```
uimpulse = inline('double (n==0)');
ustep = inline('double (n>=0)');
b= [1,0,0,1];
a = [1,-1,-1,-2];
[r, p, k] = residuez(b, a)
z = roots(b);
figure(1);
zplane (b, a);
n = [-5:10];
```

```
xn = r(1)*(p(1)).^n.*ustep(n)+r(2)*(p(2)).^n.*ustep(n)+r(3)*(p(3)).^n.*ustep(n)+k*uimp-  
pulse(n);  
figure(2);  
stem (n, xn);  
hold on ;  
xn_2 = filter(b, a, uimpulse(n));  
stem(n, xn_2, 'r');
```

3.

```
uimpulse = inline('double (n==0)');  
ustep = inline('double (n>=0)');  
b= [0,0.6,0,0];  
a = [1,-0.5,-0.25,0.125];  
[r, p, k] = residuez(b, a)  
z = roots(b);  
figure(1);  
zplane (b, a);  
n = [-5:10];  
xn =  
r(1)*(p(1)).^n.*ustep(n)+r(2)*(p(2)).^n.*(n+1).*ustep(n+1)+r(3)*(p(3)).^n.*ustep(n)+k*ui-  
mpulse(n);  
figure(2);  
stem (n, xn);  
hold on ;  
xn_2 = filter(b, a, uimpulse(n) );  
stem(n, xn_2, 'r');
```